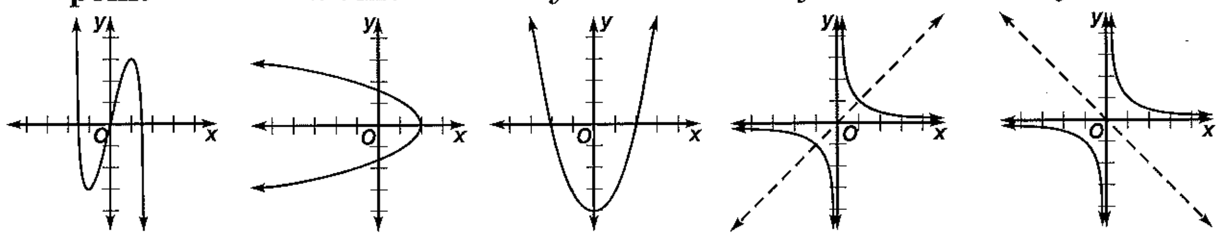
NAME ______________________ DATE ______________ PERIOD ________

3-1

Study Guide

Symmetry and Coordinate Graphs

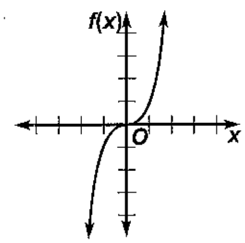
One type of symmetry a graph may have is **point symmetry.** A common point of symmetry is the origin. Another type is **line symmetry.** Some common lines of symmetry are the x-axis, the y-axis, and the lines $y = x$ and $y = -x$.



Example 1 **Determine whether $f(x) = x^3$ is symmetric with respect to the origin.**

If $f(-x) = -f(x)$, the graph has point symmetry.

Find $f(-x)$.	Find $-f(x)$.
$f(-x) = (-x)^3$	$-f(x) = -x^3$
$f(-x) = -x^3$	



The graph of $f(x) = x^3$ is symmetric with respect to the origin because $f(-x) = -f(x)$.

Example 2 **Determine whether the graph of $x^2 + 2 = y^2$ is symmetric with respect to the x-axis, the y-axis, the line $y = x$, the line $y = -x$, or none of these.**

Substituting (a, b) into the equation yields $a^2 + 2 = b^2$. Check to see if each test produces an equation equivalent to $a^2 + 2 = b^2$.

x-axis	$a^2 + 2 = (-b)^2$	*Substitute $(a, -b)$ into the equation.*
	$a^2 + 2 = b^2$	*Equivalent to $a^2 + 2 = b^2$*
y-axis	$(-a)^2 + 2 = b^2$	*Substitute $(-a, b)$ into the equation.*
	$a^2 + 2 = b^2$	*Equivalent to $a^2 + 2 = b^2$*
$y = x$	$(b)^2 + 2 = (a)^2$	*Substitute (b, a) into the equation.*
	$a^2 - 2 = b^2$	*Not equivalent to $a^2 + 2 = b^2$*
$y = -x$	$(-b)^2 + 2 = (-a)^2$	*Substitute $(-b, -a)$ into the equation.*
	$b^2 + 2 = a^2$	*Simplify.*
	$a^2 - 2 = b^2$	*Not equivalent to $a^2 + 2 = b^2$*

Therefore, the graph of $x^2 + 2 = y^2$ is symmetric with respect to the x-axis and the y-axis.

NAME ______________________ DATE ______________ PERIOD ________

3-2

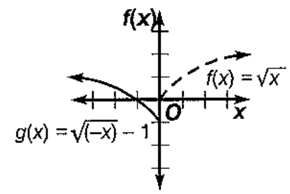
Study Guide

Families of Graphs

A **parent graph** is a basic graph that is transformed to create other members in a family of graphs. The transformed graph may appear in a different location, but it will resemble the parent graph.

A **reflection** flips a graph over a line called the *axis of symmetry*.
A **translation** moves a graph vertically or horizontally.
A **dilation** expands or compresses a graph vertically or horizontally.

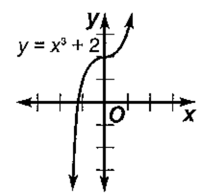
Example 1 **Describe how the graphs of $f(x) = \sqrt{x}$ and $g(x) = \sqrt{-x} - 1$ are related.**



The graph of $g(x)$ is a reflection of the graph of $f(x)$ over the y-axis and then translated down 1 unit.

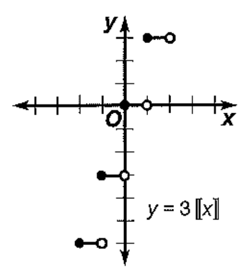
Example 2 **Use the graph of the given parent function to sketch the graph of each related function.**

a. $f(x) = x^3$; $y = x^3 + 2$



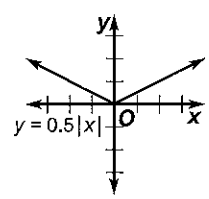
When 2 is added to the parent function, the graph of the parent function moves up 2 units.

b. $f(x) = [\![x]\!]$; $y = 3[\![x]\!]$



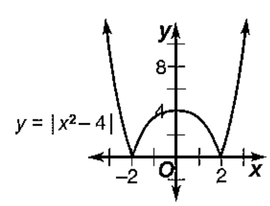
The parent function is expanded vertically by a factor of 3, so the vertical distance between the steps is 3 units.

c. $f(x) = |x|$; $y = 0.5|x|$



When $|x|$ is multiplied by a constant greater than 0 but less than 1, the graph compresses vertically, in this case, by a factor of 0.5.

d. $f(x) = x^2$; $y = |x^2 - 4|$



The parent function is translated down 4 units and then any portion of the graph below the x-axis is reflected so that it is above the x-axis.

NAME ______________________ DATE ____________ PERIOD ________

3-5 Study Guide

Continuity and End Behavior

A function is **continuous** at $x = c$ if it satisfies the following three conditions.
(1) the function is defined at c; in other words, $f(c)$ exists;
(2) the function approaches the same y-value to the left and right of $x = c$; and
(3) the y-value that the function approaches from each side is $f(c)$.

Functions can be continuous or **discontinuous.** Graphs that are discontinuous can exhibit **infinite discontinuity**, **jump discontinuity**, or **point discontinuity.**

Example 1 **Determine whether each function is continuous at the given x-value. Justify your answer using the continuity test.**

a. $f(x) = 2|x| + 3; x = 2$

(1) The function is defined at $x = 2$; $f(2) = 7$.

(2) The tables below show that y approaches 7 as x approaches 2 from the left and that y approaches 7 as x approaches 2 from the right.

x	$y = f(x)$
1.9	6.8
1.99	6.98
1.999	6.998

x	$y = f(x)$
2.1	7.2
2.01	7.02
2.001	7.002

(3) Since the y-values approach 7 as x approaches 2 from both sides and $f(2) = 7$, the function is continuous at $x = 2$.

b. $f(x) = \frac{2x}{x^2 - 1}; x = 1$

Start with the first condition in the continuity test. The function is not defined at $x = 1$ because substituting 1 for x results in a denominator of zero. So the function is discontinuous at $x = 1$.

c. $f(x) = \begin{cases} 2x + 1 & \text{if } x > 2 \\ x - 1 & \text{if } x \leq 2 \end{cases}; x = 2$

This function fails the second part of the continuity test because the values of $f(x)$ approach 1 as x approaches 2 from the left, but the values of $f(x)$ approach 5 as x approaches 2 from the right.

The **end behavior** of a function describes what the y-values do as $|x|$ becomes greater and greater. In general, the end behavior of any polynomial function can be modeled by the function made up solely of the term with the highest power of x and its coefficient.

Example 2 **Describe the end behavior of $p(x) = -x^5 + 2x^3 - 4$.**

Determine $f(x) = a_n x^n$ where x^n is the term in $p(x)$ with the highest power of x and a_n is its coefficient.

$f(x) = -x^5 \quad x^n = x^5 \quad a_n = -1$

Thus, by using the table on page 163 of your text, you can see that when a^n is negative and n is odd, the end behavior can be stated as $p(x) \to -\infty$ as $x \to \infty$ and $p(x) \to \infty$ as $x \to -\infty$.

NAME ______________________ DATE ____________ PERIOD ________

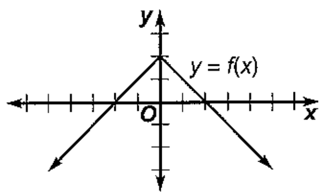
Study Guide

Critical Points and Extrema

Critical points are points on a graph at which a line drawn tangent to the curve is horizontal or vertical. A critical point may be a **maximum**, a **minimum**, or a **point of inflection.** A point of inflection is a point where the graph changes its curvature. Graphs can have an **absolute maximum**, an **absolute minimum**, a **relative maximum**, or a **relative minimum.** The general term for maximum or minimum is **extremum** (plural, *extrema*).

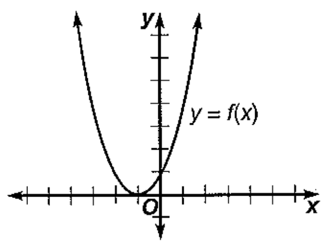
Example 1 **Locate the extrema for the graph of $y = f(x)$. Name and classify the extrema of the function.**

a.



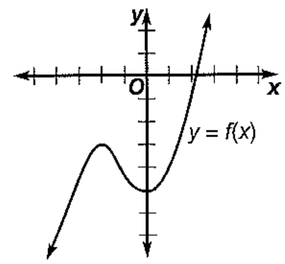
The function has an absolute maximum at (0, 2). The absolute maximum is the greatest value that a function assumes over its domain.

b.



The function has an absolute minimum at $(-1, 0)$. The absolute minimum is the least value that a function assumes over its domain.

c.



The relative maximum and minimum may not be the greatest and the least y-value for the domain, respectively, but they are the greatest and least y-value on some interval of the domain. The function has a relative maximum at $(-2, -3)$ and a relative minimum at $(0, -5)$. Because the graph indicates that the function increases or decreases without bound as x increases or decreases, there is neither an absolute maximum nor an absolute minimum.

By testing points on both sides of a critical point, you can determine whether the critical point is a relative maximum, a relative minimum, or a point of inflection.

Example 2 **The function $f(x) = 2x^6 + 2x^4 - 9x^2$ has a critical point at $x = 0$. Determine whether the critical point is the location of a maximum, a minimum, or a point of inflection.**

x	$x - 0.1$	$x + 0.1$	$f(x - 0.1)$	$f(x)$	$f(x + 0.1)$	Type of Critical Point
0	−0.1	0.1	−0.0899	0	−0.0899	maximum

Because 0 is greater than both $f(x - 0.1)$ and $f(x + 0.1)$, $x = 0$ is the location of a relative maximum.

NAME ______________________ DATE ______________ PERIOD ________

3-7 Study Guide

Graphs of Rational Functions

A **rational function** is a quotient of two polynomial functions.

The line $x = a$ is a **vertical asymptote** for a function $f(x)$ if $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$ from either the left or the right.

The line $y = b$ is a **horizontal asymptote** for a function $f(x)$ if $f(x) \to b$ as $x \to \infty$ or as $x \to -\infty$.

A **slant asymptote** occurs when the degree of the numerator of a rational function is exactly one greater than that of the denominator.

Example 1 **Determine the asymptotes for the graph of** $f(x) = \frac{2x-1}{x+3}$.

Since $f(-3)$ is undefined, there may be a vertical asymptote at $x = -3$. To verify that $x = -3$ is a vertical asymptote, check to see that $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to -3$ from either the left or the right.

x	$f(x)$
−2.9	−68
−2.99	−698
−2.999	−6998
−2.9999	−69998

The values in the table confirm that $f(x) \to -\infty$ as $x \to -3$ from the right, so there is a vertical asymptote at $x = -3$.

One way to find the horizontal asymptote is to let $f(x) = y$ and solve for x in terms of y. Then find where the function is undefined for values of y.

$$y = \frac{2x-1}{x+3}$$
$$y(x+3) = 2x - 1$$
$$xy + 3y = 2x - 1$$
$$xy - 2x = -3y - 1$$
$$x(y-2) = -3y - 1$$
$$x = \frac{-3y-1}{y-2}$$

The rational expression $\frac{-3y-1}{y-2}$ is undefined for $y = 2$. Thus, the horizontal asymptote is the line $y = 2$.

Example 2 **Determine the slant asymptote for** $f(x) = \frac{3x^2 - 2x + 2}{x - 1}$.

First use division to rewrite the function.

$$\begin{array}{r} 3x + 1 \\ x - 1 \overline{)\,3x^2 - 2x + 2} \\ \underline{3x^2 - 3x} \\ x + 2 \\ \underline{x - 1} \\ 3 \end{array} \quad \rightarrow \quad f(x) = 3x + 1 + \frac{3}{x-1}$$

As $x \to \infty$, $\frac{3}{x-1} \to 0$. Therefore, the graph of $f(x)$ will approach that of $y = 3x + 1$. This means that the line $y = 3x + 1$ is a slant asymptote for the graph of $f(x)$.